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Robust and Reduced-order Filtering: New Characterizations and Methods

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Abstract

Several challenging problems of robust filtering are addressed in this paper. First of all, for robust filtering problems, we exploit a new LMI (Linear Matrix Inequality) characterization of minimum variance or H_2 performance, and demonstrate that it allows the use of parameter-dependent Lyapunov functions while preserving tractability of the problem. The resulting conditions are less conservative than earlier techniques which are restricted to a fixed, that is not depending on parameters, Lyapunov function. The rest of the paper is focusing on the reduced-order filter problems. New LMI-based nonconvex optimization formulations are introduced for the existence of reduced-order filters. Then, several efficient optimization algorithms of local and global optimization are proposed. Nontrivial and less conservative relaxation techniques are presented as well. The viability and efficiency of the proposed tools are confirmed through computational experiments and also comparisons with earlier methods.

1 Introduction

The standard robust filter problem can be formulated as follows. Consider the uncertain linear system

$$\begin{array}{rcll} \dot{x} & = & Ax + Bw, & A \in \mathbb{R}^{n\times n} \\ y & = & Cx + Dw, & D \in \mathbb{R}^{p\times m} \\ z & = & Lx\,, & L \in \mathbb{R}^{q\times n} \end{array} \qquad (1)$$

where $x \in R^n$ is the state, $y \in R^p$ is the measured output, $z \in R^q$ is the output to be estimated and $w \in R^m$ is the zero mean white noise with identity power spectrum density matrix. The state-space data are subject to uncertainties and obey the polytopic model

$$\begin{bmatrix} A & B \\ C & D \\ L & 0 \end{bmatrix} \in \left\{ \begin{bmatrix} A(\alpha) & B(\alpha) \\ C(\alpha) & D(\alpha) \\ L(\alpha) & 0 \end{bmatrix} = \sum_{i=1}^{s} \alpha_i \begin{bmatrix} A_i & B_i \\ C_i & D_i \\ L_i & 0 \end{bmatrix}, \alpha \in \Gamma \right\}, \qquad (2)$$

where Γ is the unit simplex

$$\Gamma := \{(\alpha_1, ..., \alpha_s) : \sum_{i=1}^{s} \alpha_i = 1,\ \alpha_i \geq 0\}.$$

The problem is to find an estimator or "filter" in the form

$$\begin{array}{rcll} \dot{x}_F & = & A_F x_F + B_F y, & A_F \in \mathbb{R}^{k\times k} \\ z_F & = & L_F x_F, & L_F \in \mathbb{R}^{q\times k} \end{array} \qquad (3)$$

which provides good robust estimation in the minimum variance sense of the output z in (1). In other words, we want to minimize

$$\max_{\alpha\in\Gamma} \mathbf{E}[(z - z_F)^T(z - z_F)] \qquad (4)$$

where $\mathbf{E}$ means the mathematical expectation. Note that the expression (4) involves all possible value of the uncertainty α, hence the term robust filtering problem. When $k = n$ the filter (3) will be referred to as the full-order filter and will be termed reduced-order when $k < n$.

When all data of system (1) are exactly known, the optimal value of (4) is $Tr(LPL^T)$ and the optimal full-order solution is the well-known Kalman filter [1] defined as

$$A_F = A - B_F C, \quad B_F = PC^T(DD^T)^{-1}, \quad L_F = L\,,$$

where $P \geq 0$ is the stabilizing solution of the Riccati equation

$$AP + PA^T - PC^T(DD^T)^{-1}CP + BB^T = 0. \qquad (5)$$

Note that the existence of the stabilizing solution $P \geq 0$ of Riccati equation (5) implies that matrix A in (1) must be asymptotically stable.

An alternative solution to the full-order filter problem with exact data can be obtained by using Linear Matrix Inequality (LMI) characterizations. Indeed, rewrite (1)-(3) in compact form as

$$\begin{array}{rcl} \dot{x}_{cl} & = & \mathcal{A}x_{cl} + \mathcal{B}w, \\ z_{cl} & = & [L \quad -L_F]\, x_{cl}, \end{array} \qquad (6)$$

where

$$x_{cl} = \begin{bmatrix} x \\ x_F \end{bmatrix}, \quad \mathcal{A} = \begin{bmatrix} A & 0 \\ B_F C & A_F \end{bmatrix}, \quad \mathcal{B} = \begin{bmatrix} B \\ B_F D \end{bmatrix}, \quad z_{cl} = z - z_F. \qquad (7)$$

Then, it has been established (see e.g. [7]) that $\mathbf{E}(z_{cl}^2) < \nu$ if and only if the following matrix inequalities are feasible in the variables $\mathcal{X}$, Z, A_F, B_F and L_F

$$\begin{bmatrix} \mathcal{A}^T\mathcal{X} + \mathcal{X}\mathcal{A} & \mathcal{X}\mathcal{B} \\ \mathcal{B}^T\mathcal{X} & -I \end{bmatrix} < 0, \qquad (8)$$

$$\begin{bmatrix} \mathcal{X} & \begin{bmatrix} L^T \\ -L_F^T \end{bmatrix} \\ [L \quad -L_F] & Z \end{bmatrix} > 0, \qquad (9)$$

$$\text{Trace}(Z) < \nu. \qquad (10)$$

Thus, the problem can be formulated alternatively as

$$\min\{\nu: (8)-(10)\}. \tag{11}$$

Note that (8) is a nonlinear matrix inequality in the variables A_F, B_F and $\mathcal{X}$ because of the product terms $\mathcal{X}\mathcal{A}$. However, there are several way to reduce it to LMIs by linearizing techniques in the spirit of [11, 14] or by using the Projection Lemma in [9]. As a result, the problem can be reduced to the convex optimization problem of minimization of a linear objective over LMI constraints, an easily tractable problem with the help of currently available SemiDefinite Programming solvers.

The advantage of the proposed LMI approach, and this is an important contribution of this paper, is that it still works for the problem with unknown data as in (2) and with parameter-dependent functions $\mathcal{X} := \mathcal{X}(\alpha)$. Note that on one hand uncertainties are hardly handled with the Riccati equation approach and, parameter-dependent functions are far less conservative than customary fixed quadratic functions on the other hand.

Similarly to other results in the vein of robust control for polytopic systems, the common variable $\mathcal{X}$ has been utilized in [11, 14] for verifying (8) and (9). The resulting robust estimation may be conservative as the function $\mathcal{X}$ used for verifying (8) and (9) is fixed for all values of the parameter α. This is well-known to be a source of conservatism in applications. The issue of exploiting parameter-dependent functions $\mathcal{X}(\alpha)$ to handle problems with uncertainties is very challenging both in robust control and robust filtering. The latter issue is examined throughout this paper. We extend the results in [5] to the robust filtering problem, and derive specific linearizing transformations which lead to tractable LMI conditions for the full-order robust filtering problem with parameter-dependent functions. These results are naturally less conservative than previous ones. The latter are recovered by imposing a constant Lyapunov function ($\mathcal{X}(\alpha) := \mathcal{X}$, $\forall\alpha$) in the proposed approach.

Another challenging problem is the reduced-order filter problem which is known to be nonconvex even for the exact data case. These problems have been partially addressed e.g. in [14, 19]. Here we will propose several new solution methods for this problem. Namely, we introduce a new LMI characterization which allows us to propose several less conservative relaxations of the original problem, as well as specialized local and global optimization algorithms.

The paper is organized as follows. Robust filter problems are considered in Section 2. Section 3 is devoted to the reduced-order case with known system data and discusses various relaxations and algorithms. Finally, examples illustrating the viability and efficiency of our approach are given in Section 4.

The notations used throughout the paper is standard. M^T is the transpose of a matrix M while Trace(M) is its trace. The notation $M < 0$ ($M \leq 0$, resp.) means M is negative definite (negative semi-definite, resp.). In symmetric block matrices or long matrix expressions, we use $*$ as an ellipsis for the terms that are introduced by symmetry. A basis of the nullspace of a matrix A will be denoted $\mathcal{N}_A$.

2 Robust minimum variance filter problem

A drawback of the standard matrix inequality characterization (8)-(9) is that the function $\mathcal{X}$ used for checking the filter performance is closely interrelated with the state-space variables A_F, B_F. This makes the problem difficult to solve and causes unnecessary restrictions on the filter variables. This is particularly critical when uncertainties come into play as for polytopic systems (2). To overcome this difficulty, we exploit a Reciprocal Projection Lemma introduced in [5] to alleviate the interrelation between $\mathcal{X}$ and filter variables. This technique introduce an extra slack variable V which brings additional flexibility in the robust filtering problem. An important consequence is that the usual nonconvexity of filter synthesis problems with parameter-dependent Lyapunov functions can be bypassed. Moreover, because the function $\mathcal{X}$ is depending on uncertain parameters the resulting characterizations are generally far less conservative than customary single quadratic approaches. The following lemma will be useful in that respect. Note that for simplicity of the presentation, we shall drop the dependence of variables and data on uncertainties α for a while.

Lemma 1 *The constraints (8)-(10) are feasible in A_F, B_F, C_F,$\mathcal{X}$, Z and ν if and only if the following conditions are feasible in A_F, B_F, L_F, $\mathcal{X}$, Z, ν and V*

$$\begin{bmatrix} -(V+V^T) & V^T\mathcal{A}+\mathcal{X} & V^T\mathcal{B} & V^T \\ \mathcal{A}^TV+\mathcal{X} & -\mathcal{X} & 0 & 0 \\ \mathcal{B}^TV & 0 & -I & 0 \\ V & 0 & 0 & -\mathcal{X} \end{bmatrix} < 0, \tag{12}$$

$$\begin{bmatrix} \mathcal{X} & \begin{bmatrix} L^T \\ -L_F^T \end{bmatrix} \\ [L \quad -L_F] & Z \end{bmatrix} > 0, \tag{13}$$

$$\mathit{Trace}(Z) < \nu. \tag{14}$$

By exploiting Lemma 1, it is possible to derive tractable synthesis conditions for the robust filter problem in the full-order and reduced-rder cases.

Theorem 1 (robust full-order) *There exists a (full-order) filter such that the worst-case condition*

$$\max_{\alpha\in\Gamma} \mathbf{E}[(z-z_F)^T(z-z_F)] < \nu, \tag{15}$$

holds true, that is for all admissible systems described in (2), whenever (14) with the following (vertex) conditions hold simultaneously:

$$\begin{bmatrix} \mathcal{L}_{11}(S,V) & * & * \\ \mathcal{L}_{12}(V,\hat{X},S,\hat{A}_F,\hat{B}_F) & \mathcal{L}_{22}(\hat{X}) & * \\ \mathcal{L}_{13}(V,\hat{B}_F,S) & 0 & \mathcal{L}_{33}(\hat{X}) \end{bmatrix} < 0 \tag{16}$$

$$\begin{bmatrix} \widehat{X}_{1,i} & \widehat{X}_{3,i}^T & L_i^T \\ \widehat{X}_{3,i} & \widehat{X}_{2,i} & -\widehat{L}_F^T \\ L_i & -\widehat{L}_F & Z \end{bmatrix} > 0 \tag{17}$$

$$i = 1, \ldots, s,$$

where

$$\mathcal{L}_{11}(S,V) = \begin{bmatrix} -(V_{11}+V_{11}^T) & * \\ -(S_1+S_2) & -(S_1+S_1^T) \end{bmatrix},$$

$$\mathcal{L}_{12}(V,\hat{X},S,\hat{A}_F,\hat{B}_F) = \begin{bmatrix} A_i^T V_{11} + C_i^T \widehat{B}_F^T + \widehat{X}_{1,i} & A_i^T S_2^T + C_i^T \widehat{B}_F^T + \widehat{X}_{3,i}^T \\ \widehat{A}_F^T + \widehat{X}_{3,i} & \widehat{A}_F^T + \widehat{X}_{2,i} \end{bmatrix},$$

$$\mathcal{L}_{22}(\hat{X}) = \begin{bmatrix} -\widehat{X}_{1,i} & * \\ -\widehat{X}_{3,i} & -\widehat{X}_{2,i} \end{bmatrix},$$

$$\mathcal{L}_{13}(V,\hat{B}_F,S) = \begin{bmatrix} B_i^T V_{11} + D_i^T \widehat{B}_F^T & B_i^T S_2^T + D_i^T \widehat{B}_F^T \\ V_{11} & S_2^T \\ S_1 & S_1 \end{bmatrix}$$

$$\mathcal{L}_{33}(\hat{X}) = \begin{bmatrix} -I & * & * \\ 0 & -\widehat{X}_{1,i} & * \\ 0 & -\widehat{X}_{3,i} & -\widehat{X}_{2,i} \end{bmatrix}$$

Consequently, an upper bound of the minimum of (4) is provided by the optimization problem

$$\min_{V_{11},S_1,S_2,\widehat{A}_F,\widehat{B}_F,\widehat{L}_F,\widehat{X}_i,\nu} \nu : \begin{array}{l} (14),(16),(17), \\ i=1,2,...,s. \end{array} \tag{18}$$

The sought triple (A_F, B_F, L_F) defining the full-order filter (3) can be computed according to the following steps:

(i) *compute V_{22}, V_{21} by solving the factorization problem*

$$S_1 = V_{21}^T V_{22}^{-1} V_{21}.$$

(ii) *compute (A_F, B_F, L_F) using*

$$\begin{bmatrix} A_F & B_F \\ L_F & 0 \end{bmatrix} := \begin{bmatrix} V_{21}^{-T} & 0 \\ 0 & I \end{bmatrix} \begin{bmatrix} \widehat{A}_F & \widehat{B}_F \\ \widehat{L}_F & 0 \end{bmatrix} \begin{bmatrix} V_{21}^{-1} V_{22} & 0 \\ 0 & I \end{bmatrix}. \tag{19}$$

The Lyapunov function for confirming (15) is

$$V(x_{cl}) = x_{cl}^T [\sum_{i=1}^{s} \alpha_i \mathcal{X}_i] x_{cl}$$

with

$$\mathcal{X}_i = \begin{bmatrix} I & 0 \\ 0 & V_{21}^T V_{22}^{-T} \end{bmatrix}^{-1} \begin{bmatrix} \widehat{X}_{1,i} & \widehat{X}_{3,i}^T \\ \widehat{X}_{3,i} & \widehat{X}_{2,i} \end{bmatrix} \begin{bmatrix} I & 0 \\ 0 & V_{22}^{-1} V_{21} \end{bmatrix}^{-1}. \tag{20}$$

For the robust reduced-order relaxation see the full version [17] of this paper

3 Optimal reduced-order filter

An obvious advantage of the linearization techniques in the previous section is that it provides an accurate and practically tractable approach for the full-order robust filtering problems. It also gives a relaxation for the reduced-order case. This relaxation may however be arbitrarily conservative. The purpose of this section is to give a convenient optimization formulation for the synthesis of reduced-order filters with exact data in (2), that is $A_i := A$, $B_i := B$, etc. Even with this simplification in force, the reduced-order case is very hard because of its inherent nonconvexity. To tackle this problem, the number of complicating or nonconvex variables is reduced as much as possible. The nonconvex constraints are reformulated in such a way that they are easily handled by optimization algorithms. Local and global optimization techniques are considered at different stages of optimization process to improve efficiency. Also, new relaxation techniques are introduced.

Note that in [19] a problem related to the reduced-order filter has also been considered. Instead of the optimal kth-order filter problem, it is concerned with the kth-order filter which approximates the full-order Kalman filter. This approach provides only more or less accurate solutions to the problem with unknown degree of optimality. In this setting, an iterative algorithm has been proposed in [19] that generates a convergent sequence to a stationary point. This algorithm requires either solving multidimensional differential equations or computations involving the exponential function of matrices which are computationally consuming.

Now, our optimization formulation for the optimal reduced order filter is the following

$$\min_{X,Q,Z,\nu} \nu : (14) \tag{21}$$

$$\mathcal{N}_{[C \quad D]}^T \begin{bmatrix} XA + A^T X & XB \\ B^T X & -I \end{bmatrix} \mathcal{N}_{[C \quad D]} < 0. \tag{22}$$

$$\begin{bmatrix} X & L^T \\ L & Z \end{bmatrix} > 0. \tag{23}$$

$$Q \geq 0, \qquad \text{rank}(Q) \leq k \tag{24}$$

$$\begin{bmatrix} A^T(X-Q) + (X-Q)A & (X-Q)B \\ B^T(X-Q) & -I \end{bmatrix} < 0. \tag{25}$$

where only (24) is the source of nonconvexity. This difficulty is our main focus hereafter. When the optimal solution of (21) has been found, the optimal k-order filter (3) is easily derived by solving an LMI feasiblity problem.

For penalty/conditionla gradient and augmented Lagragian method of local optimization as well as branch and bound method of global optimization solving nonconvex optimization problem (21)-(25) we refer the reader to [2, 8, 15] and the full version [17] of the paper.

Some convexifications of (24) are also introduced here basing on the following result.

Lemma 2 *A positive semi-definite matrix Q of dimension $n \times n$ has a rank less than $k \leq n$ if it has at least $(n-k)$ zero diagonal entries, i.e. there are indexes $1 \leq i_1 < i_2 < ... < i_{n-k} \leq n$ such that $Q_{i_j i_j} = 0$, $j = 1, 2, ..., (n-k)$.*

From the above lemma, it follows that for any $1 \leq i_1 < i_2 < ... < i_{n-k} \leq n$ an upper bound for (21) is provided by the following (convex) LMI optimization problem

$$\text{(RL)} \quad \min_{X,Q,Z,\nu} \nu : \begin{array}{l} (10),(22),(23),(25),\ Q \geq 0 \\ Q_{i_j i_j} = 0,\ j = 1,2,...,(n-k) \end{array} \quad (26)$$

Clearly, when either $k = n$ (full order case) or $k = 0$ then (21) is equivalent to (26), i.e. (21) becomes a (convex) LMI optimization problem.

4 Illustrative Examples

This section discusses some examples and provide comparison results with earlier techniques both for robust and reduced-order filtering problems..

4.1 Robust filter examples

We consider the following example borrowed from [11, (68)-(70)]

$$\begin{aligned} \dot{x} &= \begin{bmatrix} 0 & -1+0.3\alpha \\ 1 & -0.5 \end{bmatrix} x + \begin{bmatrix} -2 & 0 \\ 1 & 0 \end{bmatrix} w \\ y &= \begin{bmatrix} -100+10\beta & 100 \end{bmatrix} x + \begin{bmatrix} 0 & 1 \end{bmatrix} w \\ z &= \begin{bmatrix} 1 & 0 \end{bmatrix} x \end{aligned} \quad (27)$$

with two alternative uncertainty set descriptions, either

$$|\alpha| \leq 1, \quad |\beta| \leq 1, \quad (28)$$

or

$$|\alpha| \leq 1, \quad \alpha = \beta. \quad (29)$$

The comparison between results obtained using the present results of the paper and those of [11, 14] are provided in Table 1. Computations were performed using the Matlab LMI Control Toolbox [10]. From this

method	system	filter order	BUB
[11]	(27),(28)	full	5.728
[14]	(27),(28)	full	4.867
present	(27),(28)	full	2.382
[11]	(27),(29)	full	4.819
[14]	(27),(29)	full	4.373
present	(27),(29)	full	2.382
[14]	(27),(28)	1	4.946
present	(27),(28)	1	3.001
[14]	(27),(29)	1	4.556
present	(27),(29)	1	3.079

Table 1: computational comparisons
robust full- and reduced-order filters

Table, the advantage of the proposed method appears clearly. Note that with all α satisfying (29), the asymptotic stability of $A(\alpha)$ in (27) can be checked by a single Lyapunov function $V(x) = x^T X x$. However, if we replace (29) with

$$|\alpha| \leq 3,\ \beta| \leq 1 \quad (30)$$

then a single Lyapunov function is not satisfactory for checking the asymptotic stability, eventhough $A(\alpha)$ in (27) is asymptotically for all $|\alpha| \leq 3.2$. As a result, the approaches of [11, 14] with parameter-independent Lyapunov functions cannot work (LMI constraints are infeasible). In contrast, the techniques of Theorems 1 is still valid in this case. The computational results for (27), (30) and also (27) with

$$|\alpha| \leq 3,\ \beta = \alpha \quad (31)$$

are sketched in Table 2.

method	system	filter order	best upper bound
[11] or [14]	(27),(30)	full	$+\infty$
present	(27),(30)	full	93.365
present	(27),(30)	1	106.493
[11] or [14]	(27),(31)	full	$+\infty$
present	(27),(31)	full	100.963
present	(27),(31)	1	106.517

Table 2: Further computational comparisons
robust full- and reduced-order filters

4.2 Reduced-order examples with exact data

Consider the following system borrowed from [19, (5.4)-(5.6)] for system (1) with

$$A = \begin{bmatrix} 0 & 1.0 & 0.5 \\ -5.0 & -0.02 & 0 \\ 1.5 & 0 & -0.1 \end{bmatrix},\ B = \begin{bmatrix} 0 & 0 \\ 1 & 0 \\ 1 & 0 \end{bmatrix},$$
$$C = \begin{bmatrix} 1 & 1 & -2 \end{bmatrix}, D = \begin{bmatrix} 0 & 1 \end{bmatrix},$$
$$L = \begin{bmatrix} 1 & 1 & -2 \end{bmatrix}.$$

At the reduced 2nd-order case, the best value given in [19], is $\sqrt{\nu} = 4.74$. After a few iterations, the penalty/conditional gradient method achieves the much better value $\sqrt{\nu} = 2.4503$ corresponding to

$$Q = \begin{bmatrix} 2.3521 & 0.4489 & -1.4617 \\ 0.4489 & 0.3310 & -0.5034 \\ -1.4617 & -0.5034 & 1.1137 \end{bmatrix}.$$

The later value is very near to the true global optimal value $\sqrt{\nu} = 2.4253$ found by BB method which corresponds to

$$Q = \begin{bmatrix} 2.5425 & 0.4700 & -1.6467 \\ 0.4700 & 0.3211 & -0.5210 \\ -1.6467 & -0.5210 & 1.2668 \end{bmatrix}.$$

Consider another example from [19, (5.4)-(5.6)] for (1) with

$$A = \begin{bmatrix} 0 & -0.1 & 0 & 0 & 0 \\ 1 & -0.3 & 0 & 0 & 0 \\ 0 & -0.2 & 0 & 0 & 0.016 \\ 0 & -0.3 & 1 & 0 & 0.06 \\ 0 & -0.1 & 0.1 & -1.5 & -0.9 \end{bmatrix},$$

$$B = \begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix},$$

$$C = \begin{bmatrix} 0.1 & 0 & 0 & -0.5 & 1.6 \\ 0.1 & 0.2 & 0 & -0.3 & 0.12 \end{bmatrix},$$

$$D = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix},$$

$$L = \begin{bmatrix} 0.1 & 0 & 0 & -0.5 & 1.6 \\ 0.1 & 0.2 & 0 & -0.3 & 0.12 \end{bmatrix}.$$

The best result of [19] gives $\sqrt{\nu} = 2.06$ for reduced 3rd-order filters. Using the relaxation (26) yields the improved value $\sqrt{\nu} = 1.9120$ corresponding to

$$Q = \begin{bmatrix} \mathbf{0} & -0.0008 & -0.0001 & -0.0016 & -0.0136 \\ -0.0008 & 0.1183 & 0.0002 & 0.0563 & 0.0763 \\ -0.0001 & 0.0002 & \mathbf{0} & -0.0012 & 0.0115 \\ -0.0016 & 0.0563 & -0.0012 & 0.6451 & 0.5659 \\ -0.0136 & 0.0763 & 0.0115 & 0.5659 & 2.1510 \end{bmatrix}.$$

Note that this value is almost globally optimal for the nonconvex problem (21) since it is very close to the full-order case, $\sqrt{\nu} = 1.77$.

5 Concluding remarks

In this paper, different techniques and tools for robust and/or reduced-order minimum variance filter problems have been developed. For the synthesis of robust filters, we introduce a new LMI representation which allows the use of parameter-dependent Lyapunov functions while preserving tractability of the problem. This approach generalizes and improves on earlier techniques.

For the reduced-order synthesis, we have introduced more or less conservative relaxations. These relaxed formulations are readily solved as LMI programs but might fail to achieve satisfactory performance levels. In such case, one can either use a penalty/conditional gradient algorithm to get a better local solution or a combination of the penalty/conditional gradient method and the BB method if global optimality is practically demanded.

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