Problem Set 2
ECE 257A: Multi-User Communications and Networking
Spring 2007

Problem Set. (Due Nov 3, 07)

1. Prove that the Euclidean norm on $\mathbb{R}^n$ is convex. How about the Euclidean distance defined on $\mathbb{R}^n \times \mathbb{R}^n$?

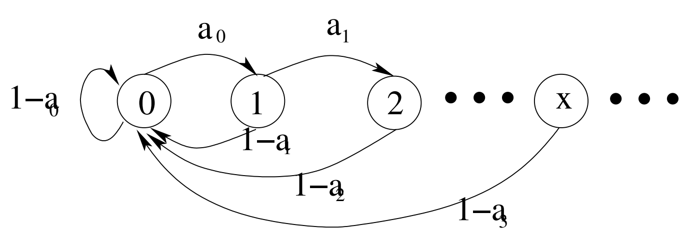
2. Consider a convex function f from $\mathbb{R}^n$ to $\mathbb{R}^m$. For any convex function on $\mathbb{R}^m$, let's say g, define function g_f on $\mathbb{R}^n$ as

$$g_f(x) = g(f(x)) \qquad \forall x \in \mathbb{R}^n.$$

   When is g_f convex?

   Hint: Put a condition on g to guarantee convexity of g_f for any convex f.

3. Consider a discrete time MC on the non-negative integers such that from x, the chain goes to $x+1$ with probability $0 < a_x < 1$ and to 0 with probability $1 - a_x$.



   - Prove that the chain is irreducible.
   - Find the necessary and sufficient conditions (on the sequence $\{a_x\}_0^\infty$) for this chain to be transient, null recurrent, an positive recurrent.

4. Consider the following game. You can buy a ball to throw at a target for a dollar. Your chances of hitting the target is a half. If you hit the target you are given another ball. When you run of balls you loose the game. While if you collect 3 balls you win a stuffed animal. (a) What is the probability of winning the toy? (b) What is the expected number of plays you need to win or lose the game? (c) Can you generalize this when instead of three you are required to collect N balls? *Hint:* First draw a state transition diagram for this game. Then Find the recursive equations for $E(N|X_0 = 1)$ and $E(N|X_0 = 2)$; note that $E(N|X_0 = 0) = E(N|X_0 = 3) = 0$. This type of approach to finding expectation and distribution of stopping times in Markov Chains is called one-step analysis.

5. Due to complexity of maximum matching algorithms, some researchers have proposed practical sub-optimal algorithms which are calculated in three steps. One such algorithm was *Round-Robin matching*, RRM, algorithm seen in your HWs. Another example is *Parallel Iterative matching*, PIM, algorithm.

   PIM uses randomness to avoid synchronization problems we saw with RRM. Each input and each output is equipped with a random scheduler. The algorithm consists of the following three steps:

Step 1 *Request:* Each input sends a request to every output for which it has a queued cell,

Step 2 *Grant:* If an output receives any requests, it grants to one by randomly selecting a request uniformly among all requests.

Step 3 *Accept:* If an input receives a grant, it accepts one by randomly selecting an output uniformly among those that granted this input.

(a) Construct an example to show that an PIM algorithm can result in a matching which is not of maximum size.

(b) Consider an $N \times N$ switch. Assuming all VOQs have a non-zero backlog, calculate the probability that an input remains ungranted.

(c) Use (b) to argue that as number of ports (N) increases throughput is strictly below 1.