Topic 1

ECE 257A: Multi-User Communications and Networking

Fall 2007

Problem Set. 1 (Due October 18th, 2007)

1. Show that in the sense of Lagrange Duality Theorem, the dual to the linear programming problem

$$\begin{aligned} \min_x \quad & b^T x \\ \textit{subject to} \quad & Ax \geq c, \\ & x \geq 0, \end{aligned}$$

   where x and b are n dimensional vectors, c is an m dimensional vector, and A is an $m \times n$ matrix, is another linear program:

$$\begin{aligned} \max_\lambda \quad & c^T \lambda \\ \textit{subject to} \quad & A^T \lambda \leq b, \\ & \lambda \geq 0, \end{aligned}$$

2. Show that the sum of two convex mappings is also convex. Can you say this about the maximum of two convex mappings? How about the minimum?

3. Consider vector p and scalar $\alpha \neq 1$. The notion of the proportional fairness can be generalized to (p, α)-proportional fairness in the following sense: A vector of rates x is (p, α)-proportionally fair if it is feasible ($x^* \in \Delta$) and for any other feasible vector x

$$\sum_i p_i \frac{x_i - x_i^*}{(x_i^*)^\alpha} \leq 0.$$

   Show that a vector is (p, α)-proportionally fair if and only if it solves the network utility problem for $g(x) = \sum_i p_i f_\alpha(x_i)$ where $f_\alpha(y) := \frac{x^{1-\alpha}}{(1-\alpha)}$.

   You need the following definition:

Definition 1 *A set* $\mathcal{X} \in \mathbb{R}^n$ *is said to have the solidarity property iff for* $\forall x := (x_1, \ldots, x_n) \in \mathcal{X}$*,* $\exists \epsilon > 0$*, such that for* $\forall \alpha \leq \epsilon$ *and* $\forall i$ *,* $\exists \delta > 0, (\delta \leq \epsilon)$ *such that* $\forall j, x - \alpha e_i + \delta e_j \in \mathcal{X}$*.*

4. Consider a Network with n flows with rates $R_1, R_2, \ldots, R_n$. Assume the feasible set of rates is identified with $\Delta \in \mathbb{R}^n$ such that Δ has the solidarity property. Show that the maxmin fair solution in this network, $(R_1^*, \ldots, R_n^*)$, is unique and $R_i^* = R_j^*$?